WS #4-7
Compound Interest

1. You will be responsible to read the section completely and review the definitions and applications.
   - A. Simple interest
   - B. Pay Periods
   - C. Compound interest
   - D. Continuous Interest
   - E. Effective rate of interest
   - F. Present Value

2. A credit union pays interest of 8% per annum compounded quarterly on a certain savings plan. If $100 is deposited in such a plan and the interest is left to accumulate, how much will be in the account after 1 year? $A = P\cdot(1+\frac{r}{n})^{nt} \rightarrow A = 1000(1+\frac{.08}{4})^{4\cdot 1} = \boxed{\$1082.43}$

3. Compound and continuous interest formulas

   A. Compound Interest formula : $A = P\cdot(1+\frac{r}{n})^{nt}$

   Investing $1000 at an annual rate of 10% compounded annually, semiannually, quarterly, monthly and daily yields the following amounts after 5 year:

   1. Annually $\rightarrow 1000(1+.1/1)^{1(5)} = \boxed{\$1610.51}$
   2. Semiannually $\rightarrow 1000(1+.1/2)^{2(5)} = \boxed{\$1628.89}$
   3. Quarterly $\rightarrow 1000(1+.1/4)^{4(5)} = \boxed{\$1638.62}$
   4. Monthly $\rightarrow 1000(1+.1/12)^{12(5)} = \boxed{\$1645.31}$
   5. Daily $\rightarrow 1000(1+.1/365)^{365(5)} = \boxed{\$1648.61}$

   B Continuous Interest formula : $A = Pe^{rt}$

   The amount yielded ~~fro~~ for continuous compounding for 5 year is: $A = 1000e^{.1(5)} = \boxed{\$1648.72}$

4. Effective rates of return

   On January 2, 2004, $2000 is placed in an Individual Retirement Account (IRA) that will pay interest of 7% per annum compounded continuously.
   - A. What will the IRA be worth on January 1, 2024? $A = Pe^{rt} \rightarrow A = 2000e^{(.07)(20)} = \boxed{\$8110.40}$
   - B. What is the effective annual rate of interest?

   ① Compute interest earned on \$2,000 @ 7% compounded continuously for 1 yr. $\rightarrow A = 2000e^{(.07)(1)} = \$2145.02$

   ② interest earned = 2145.02 − 2000 = \$145.02

   $I = prt \rightarrow \frac{145.02}{2000} = \frac{2000r\cdot 1}{2000}$

   $0.07251 = r \rightarrow \boxed{r = 7.251\%}$

5. Present Value

   $P = A\cdot(1+\frac{r}{n})^{-nt}$

   $P = Ae^{-rt}$

   A zero-coupon bond (non-interesting-bearing) bond can be redeemed in 10 years for $1000. How much should you be willing to pay for it now if you want a return of
   - A. 8% compounded monthly? $\rightarrow P = 1000(1+\frac{.08}{12})^{-12(10)} = \boxed{\$450.52}$
   - B. 7% compounded continuously? $\rightarrow P = 1000e^{-.07(10)} = \boxed{\$496.59}$

6. What rate of interest compounded annually should you seek if you want to double your investment in 5 years? If P = principal & we want to Double P, the Amount will be 2P

   $A = P\cdot(1+\frac{r}{n})^{nt}$

   $2P = P(1+\frac{r}{1})^{1(5)}$

   $2 = (1+r)^5$

   $\sqrt[5]{2} = 1+r$

   $r = \sqrt[5]{2} - 1$

   $r = .148698$

   $\boxed{r \approx 14.87\%}$

7A. How long will it take for an investment to double in value if it earns 5% compounded continuously?

   $2P = Pe^{.05t} \rightarrow 2 = e^{.05t} \rightarrow \ln 2 = \ln e^{.05t} \rightarrow \frac{\ln 2}{.05} = \frac{.05t}{.05} \rightarrow \boxed{t \approx 13.86}$

B. How long will it take to triple in value?

   $3P = Pe^{.05t} \rightarrow 3 = e^{.05t} \rightarrow \ln 3 = \ln e^{.05t} \rightarrow \frac{\ln 3}{.05} = \frac{.05t}{.05} \rightarrow t \approx 21.97 \rightarrow \boxed{t = 22 \text{ yrs}}$